Indices or Powers

The Powers of 2

$2^5 = 1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1024$
$2^4 = 1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 512$
$2^3 = 1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$
$2^2 = 1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$
$2^1 = 1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$
$2^5 = 1 \times 2 \times 2 \times 2 \times 2 \times 2 = 32$
$2^4 = 1 \times 2 \times 2 \times 2 \times 2 = 16$
$2^3 = 1 \times 2 \times 2 \times 2 = 8$
$2^2 = 1 \times 2 \times 2 = 4$
$2^1 = 1 \times 2 = 2$
$2^0 = 1 = 1$
$2^{-1} = 1 \div 2 = 0.5$
$2^{-2} = 1 \div 2 \div 2 = 0.25$
$2^{-3} = 1 \div 2 \div 2 \div 2 = 0.125$
$2^{-4} = 1 \div 2 \div 2 \div 2 \div 2 = 0.0625$
$2^{-5} = 1 \div 2 \div 2 \div 2 \div 2 \div 2 = 0.03125$

Notice how each number halves as you work your way down to 2^0, and beyond.

Index Laws

These are special rules about indices that make our life a lot easier, especially when it comes to using them in algebra.

When two powers with the same base are divided, the indices are subtracted.

$$\frac{10^9}{10^5} = 10^{9-5} = 10^4$$

$$a^b \div a^c = a^{b-c}$$

When two powers with the same base are multiplied, the indices are added.

$$3^2 \times 3^4 = 3^{2+4} = 3^6$$

$$a^b \times a^c = a^{b+c}$$

$$\left(4^2\right)^3 = 4^{2\times 3} = 4^6$$

When a power is raised to another power, the indices are multiplied.

$$(a^b)^c = a^{b \times c}$$

The root of a number gives the same answer as the inverse index of the number.

$$\sqrt[3]{27} = 27^{\frac{1}{3}}$$

$$\sqrt[b]{a} = a^{\frac{1}{b}}$$

Anything raised to the power 0 is 1.

$$250^0 = 1$$

$$a^0 = 1$$

A negative index gives the same answer as the inverse of the positive index.

$$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$$

$$a^{-b} = \frac{1}{a^b}$$

Useful Powers of 10

$10^{1\ \text{Googol}}$		= 1 Googolplex
10^{100}		= 1 Googol
10^{12}	= 1 000 000 000 000	= 1 Trillion
10^9 =	1 000 000 000	= 1 Billion
10^6 =	1 000 000	= 1 Million
10^3 =	1 000	= 1 Thousand
10^2 =	100	= 1 Hundred
10^1 =	10	= Ten
10^0 =	1	= One
10^{-1} =	0.1	= 1 Tenth
10^{-2} =	0.01	= 1 Hundredth

Challenge

What is the largest number you can make with these digits?

1 2 3 4